

How degeneracies can obscure interesting physics

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We show how degeneracies, accidental or otherwise, can obscure some interesting physics. We further show how one can get around this problem.

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In a 2006 publication [1] Escuderos and Zamick found some interesting behaviour in the $g_{9/2}$ shell. Unlike the lower shells, e.g. $f_{7/2}$, seniority is not a good quantum number in the $g_{9/2}$ shell. Despite this, it was found that in a matrix diagonalization with four identical particles in the $g_{9/2}$ shell with total angular momentum $I = 4$ or 6 , one unique state emerged no matter what interaction was used. Before the mixing one has two states with seniority $v = 4$ and one with $v = 2$. The surprise was that, after the diagonalization, one gets a unique state that is always the same independently of the interaction used. This unique state has seniority $v = 4$. The components of the wave function are given in the third column after $[J_p, J_n]$. The problem to be dealt with was not only why this state did not mix with the $v = 2$ state but also why it does not mix with the other $v = 4$ state. But this will not concern us here. Rather we will use this as an example of how degeneracies can obscure interesting physics.

It was already commented on in the 2007 paper [2] that cfp's for identical particles are usually calculated using a pairing interaction. With such an interaction, the two $v = 4$ states are degenerate, i.e. they have the same energy. This means that any linear combination of the two states can emerge in a matrix diagonalization. One can get different combinations with different programs or even with the same program run at different times. Thus the emergence of a unique state gets completely lost.

In this work we consider a less obvious example: a matrix diagonalization of two proton holes and two neutron holes in the $g_{9/2}$ shell, i.e. we consider ^{96}Cd rather than ^{96}Pd , the latter consisting of four proton holes (whether we consider holes or particles does not matter). We use a quadrupole-quadrupole interaction $Q \cdot Q$ for the matrix diagonalization. The two-body matrix elements in units of MeV from $J = 0$ to $J = 9$ are: $-1.0000, -0.8788, -0.6516, -0.3465, -0.0152, 0.2879, 0.4849, 0.4849, 0.1818$, and -0.5454 .

We show the results in Table I. For $I = 4$ we get 14 eigenfunctions, but we list only two of them in the first two columns. The reason we single these out is that they are degenerate—both are at an excitation energy of 3.5284 MeV.

In the third wave function column we have the unique state, one that emerges, as we said above, with any interaction, however complicated, e.g. CCGI [3]. But now we have to modify the phrase “any interaction”. We do not see this unique state when we use the $Q \cdot Q$ interaction—none of the 14 states looks like the one in column 3. Learning from our experience with the pairing interaction, we suspect that the problem lies with the two degenerate states at 3.5284 MeV. We assumed that the two states were mixtures of one $T = 0$ and one $T = 2$ state.

Table I: Selected $I = 4^+$ states in ^{96}Cd with a $(g_{9/2})^4$ configuration. On the second row we give the energies in MeV.

$[J_p, J_n]$	Mix $T = 0, 2$ 3.5284	Mix $T = 0, 2$ 3.5284	$T = 2, v = 4$ unique 6.5285	$T = 0$ untangled 3.5284	other $T = 2, v = 4$
$[0, 4]$	0.0000	0.0000	0.0000	0.0000	0.0000
$[2, 2]$	-0.3250	-0.4170	-0.4270	0.3123	-0.0255
$[2, 4]$	-0.2364	-0.2472	-0.2542	0.2289	-0.1986
$[2, 6]$	0.2168	0.3043	0.3107	-0.2076	-0.1976
$[4, 4]$	0.0207	0.2390	0.2395	-0.0135	-0.3313
$[4, 6]$	-0.1826	-0.1364	-0.1418	0.1784	0.2245
$[4, 8]$	0.0934	0.1540	0.1567	-0.0888	0.3874
$[6, 6]$	-0.1312	0.1678	0.1638	0.1362	0.5645
$[6, 8]$	-0.1343	0.0357	0.0316	0.1353	0.0247
$[8, 8]$	-0.7421	0.5881	0.5625	0.7594	-0.1087

We can remove the degeneracy without altering the wave functions of the non-degenerate states by adding a $t(1) \cdot t(2)$ interaction to the Hamiltonian. This will shift energies of states of different isospin. What we actually did was equivalent to this. We added -1.000 MeV to the two-body $T = 0$ matrix elements. These had odd spin

$J = 1, 3, 5, 7, 9$. What emerged is shown in columns 3 and 4. The degeneracy is removed. We have a $T = 2$ state in the third column shifted up by 3 MeV and in the fourth column a $T = 0$ state unshifted. The wave function components are different from what they are in the first two columns. The $T = 2$ state is the unique state we were talking about—one that emerges with any interaction, e.g. CCGI or delta. It is the double analog of a state of four identical proton holes (^{96}Pd). The untangled $T=0$ state in the 4'th column has vanishing $[0,4]$ and $[4,0]$ components. Unlike the wave function in the 3'rd column this wave function does not appear as an eigenstate for most other interactions.

In the last column, we list the other $T = 2, v = 4$ state. One sees this on the list when one uses a seniority-conserving interaction such as a delta interaction. However, for a general interaction, it does not appear. This is because it gets mixed with the $T = 2, v = 2$ state. Only the state in the third column remains unscathed when we turn on some arbitrary interaction—and only it does not end up being degenerate with some other state.

There is also a unique $v=4 J=6^+$ state. With the pairing interaction this is degenerate with another $v=4 J=6^+$ state and so the uniqueness gets obscured. However with the Q.Q interaction, unlike the case for $J=4^+$, this unique $v=4 T=2 J=6^+$ state is not degenerate with another state. Hence, even with Q.Q this state will appear in a calculation.

There are other examples of confusions. The electric dipole moment of the neutron would vanish if parity conservation holds. But at a more important level, it vanishes if time reversal invariance holds.

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